Effect Size

Effect size is a quantitive measure of the strength of a phenomenon and is often used as part of a meta-analysis. We will be looking at two measures of effect size, standardised mean difference and mean difference. For each type of effect size, a larger absolute value always indicates a stronger effect.

1. Mean difference

Use this measure for effect size when all the studies use the same scale of measurement. Say you're reading a study and it reports the mean for two groups (treatment and control) and suppose you want to compare these two means. Let μ_1 and μ_2 be the two population means of the two groups. The population mean difference is defined as

$$\Delta = \mu_1 - \mu_2.$$

But you will rarely know the population means so we will describe how to estimate this number. When we have two independent samples we can estimate the following steps

(1) Let $\overline{X}_1$ and $\overline{X}_2$ be the two sample means, the sample sizes are n_1 and n_2 and S_1 and S_2 are the sample standard deviations.

(2) The sample estimate, D, of Δ is just the difference between the sample means, that is

$$D = \overline{X}_1 - \overline{X}_2.$$

(3) If we assume both samples have the same population standard deviation then the variance of D is

$$V_D = \frac{n_1 + n_2}{n_1 n_1} S_C^2,$$

where

$$S_C = \sqrt{\frac{(n_1 - 1) S_1^2 - (n_2 - 1) S_2^2}{n_1 + n_2 - 2}}.$$

If we don't assume the population variance is the same then

$$V_D = \frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}$$

(4) Either way the standard error is

$$SE_D = \sqrt{V_D}.$$

Example 1.1. For example, suppose that a study has sample means $\overline{X}_1 = 103.00$, $\overline{X}_2 = 100.00$, sample standard deviations $S_1 = 5.5$, $S_2 = 4.5$, and sample sizes $n_1 = n_2 = 50$. The mean difference D

$$D = 103.00 - 100.00 = 3.00$$

If we assume both samples have the same population standard deviation then

$$S_C = \sqrt{\frac{(50-1) \times 5.5^2 + (50-1) \times 4.5^2}{50 + 50 - 2}} = 5.0249$$

So the variance would equal

$$V_D = \frac{50 + 50}{50 \times 50} \times 5.0249 = 1.01$$

And $SE_D = \sqrt{1.01} = 1.005$. However if we don't make that assumption

$$V_D = \frac{5.5^2}{50} + \frac{4.5^2}{50} = 1.01, \text{ and } SE_D = 1.005$$

2. Standardised Mean Difference

Use this in the same situation as the **mean difference** except that this time there may be a different scale of measurement used. The Standard mean difference is also known as Cohen's d. In this case we divide the mean difference D by the sample standard deviation in each case. This will then create an index which is comparable across studies and measurements. If μ_1 and μ_2 are the two population means associated with the samples and the both have population standard deviation σ then the standard mean difference is

$$\delta = \frac{\mu_1 - \mu_2}{\sigma}$$

We can estimate δ using the sample means $\overline{X}_1$ and $\overline{X}_2$ of two independent groups.

$$d = \frac{\overline{X}_1 - \overline{X}_2}{S_w},$$

where S_w is the standard deviation within each group.

$$S_w = \sqrt{\frac{(n_1 - 1)\, S_1^2 + (n_2 - 1)\, S_2^2}{n_1 + n_2 - 2}},$$

and n_1 and n_2 are the size of each sample.

The variance of d is given by

$$V_d = \frac{n_1 + n_2}{n_1 n_2} + \frac{d^2}{2\,(n_1 + n_2)},$$

and the standard error is

$$SE_d = \sqrt{V_d}$$

d	Percentage of control fall below average of experimental
0.0	50%
0.2	58%
0.5	69%
0.8	79%
1.0	84%
1.4	92%

An effect size of 0.2 is said to be ``$small$", $0.5$ is ``$medium$" and 0.8 is said to be ``$large$".

Example 2.1. For example, suppose that a study has sample means $\overline{X}_1 = 103.00$, $\overline{X}_2 = 100.00$, sample standard deviations $S_1 = 5.5$, $S_2 = 4.5$, and sample sizes $n_1 = n_2 = 50$. To calculate the standard mean difference we begin by calculating standard deviation within the group.

$$S_w = \sqrt{\frac{(50-1) \times 5.5^2 + (50-1) \times 4.5^2}{50 + 50 - 2}} = 5.0249$$

$$d = \frac{103.00 - 100.00}{5.0249} = 0.5970$$

$$V_d = \frac{50+50}{50\times 50} + \frac{0.5970^2}{2\,(50+50)} = 0.0418$$

$$SE_d = \sqrt{0.0418} = 0.2044.$$

A measure of effect size, together with confidence intervals, are often used to calculate forest plots. The measure of effect size in your forest plot is typically just mean of the effect sizes from each individual study.